LEVEL 6 (Calculus) BLITZ 2018

Directions:
1. Select the most correct answer for each questions and mark it on you answer form.
2. No calculators of any sort are allowed.

1) The average value $f(x) = \tan(x)$ on the interval from $x = 0$ to $x = \frac{\pi}{3}$ is

a) $\ln\frac{1}{2}$ b) $\frac{3}{\pi}\ln 2$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{9}{\pi}$ e) none of these

2) If $f(x) = x^2 + 2x$, then $\frac{d}{dx}\left(f(\ln x)\right) =$

a) $\frac{2\ln x + 2}{x}$ b) $2x\ln x + 2$ c) $2\ln x + \frac{2}{x}$

d) $2\ln x + 2$ e) none of these

3) The line $x + y = k$, where k is constant, is the tangent to the graph of $y = x^2 + 3x + 1$ The value of k is

a) 1 b) -1 c) -2 d) -3 e) none of these

4) $\lim\limits_{x\to 2^-} \frac{|x-2|}{x-2}$ is

a) $-\infty$ b) -1 c) ∞ d) does not exist e) none of these

5) If $f(x) = (x-1)(x^2+2)^3$, then $f'(0) =$

a) 0 b) -4 c) 4 d) 8 e) none of these

6) If $\sin(xy) = x$, then $\frac{dy}{dx} =$

a) $\frac{1}{x\cos(xy)}$ b) $\frac{1-\cos(xy)}{\cos(xy)}$ c) $\frac{1-y\cos(xy)}{x\cos(xy)}$

d) $\frac{y(1-\cos(xy))}{x}$ e) none of these

7) Find dy/dx for the curve $xy^2 - 2y + 4y^3 = 6$ at the point where $y = 1$.

a) $-\frac{1}{18}$ b) $-\frac{1}{26}$ c) $\frac{15}{18}$ d) $-\frac{11}{18}$ e) none of these

8) If f be a function with a second derivative given by $f''(x) = x^4 - 9x^3 + 18x^2$, then the $x-$coordinate(s) of the points of inflection of the graph of f is (are)

a) 0 only b) 3 and 0 only c) 3 and 6 only d) 0, 3, and 6

e) none of these

9) The number of vertical tangents to the graph of $y^2 = x - x^3$ is

a) 3 b) 2 c) 1 d) 0 e) none of these

10) $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}}\,dx =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{6} - 1$ c) $1 - \frac{\sqrt{3}}{2}$ d) $2 - \sqrt{3}$ e) none of these

11) If f is a twice differentiable function with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the tangent line to the graph of f at $x = 2$?

a) 0.6 b) 0.7 c) 1.3 d) 1.4 e) none of these

12) The velocity of a particle in motion along a line for $t \geq 0$ is $v(t) = \ln(2 - t^2)$. The acceleration when the object is at rest is

a) -2 b) 0 c) $\frac{1}{2}$ d) 1 e) none of these

13) A particle moves along the axis with velocity given by $v(t) = 3t^2 + 6t$ for $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

a) 4 b) 6 c) 9 d) 12 e) none of these

14) The region in the first quadrant bounded by the curve $y = \sec x$, $x = \frac{\pi}{4}$ and the axes is rotated about the $x-$axis. The volume of the solid generated is

a) $\frac{\pi^2}{4}$ b) $\pi - 1$ c) π d) 2π e) none of these

15) If $f(x) = \frac{x^2 + x}{x}$ where $x \neq 0$ and $f(0) = 1$ and

I. f is defined at $x = 0$
II. $\lim_{x \to 0} f(x)$ exists
III. f is continuous at $x = 0$

then the following statements is (are) true

a) I only b) II only c) I and II only

d) I, II, and III e) none of these

16) The area bounded by the parabola $y = x^2$ and the lines $y = 1$ and $y = 9$ is

a) $\frac{20}{3}$ b) $\frac{52}{3}$ c) 36 d) $\frac{104}{3}$ e) none of these

17) If $\int_1^2 f(x - c)\, dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\, dx =$

a) 5 b) -5 c) $c - 5$ d) $5 - c$ e) none of these

18) $\int_0^1 (x+1)\, e^{x^2+2x}\, dx =$

a) $\frac{1}{2}e^3 - 1$ b) $\frac{e^3-1}{2}$ c) $2(e^3-1)$ d) $2e^3-1$ e) none of these

19) $\lim_{\theta \to 0} \frac{sin^2\theta}{\theta}$

a) 0 b) 1 c) 2 d) 1/2 e) none of these

20) The number of tangent lines passing through the point (1, 2) to the curve $y = \frac{x}{x+1}$

a) 0 b) 1 c) 2 d) 3 e) none of these

21) $\lim_{h \to 0} \frac{e^{h+2} - e^2}{h}$ is

a) 0 b) 1 c) e d) e^2 e) none of these

22) $\lim_{x \to 0} \frac{tan3x}{3tan2x}$

a) 0 b) 1/3 c) 2/3 d) 2 e) none of these

23) If $f^{-1}(x)$ is the inverse of $f(x) = 2e^{-x}$, then $f^{-1}(x) =$

a) $\ln\left(\frac{2}{x}\right)$ b) $\ln\left(\frac{x}{2}\right)$ c) $\frac{1}{2}\ln x$ d) $\sqrt{lnx}$

e) none of these

24) $\lim_{h \to 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right) =$

a) 0 b) 1 c) $\frac{1}{2}$ d) e^2 e) none of these

25) The general solution of the differential equation $\frac{dy}{dx} = y$ is a family of

a) parabolas b) straight lines c) hyperbolas

d) ellipses e) none of these

26) $\frac{d}{dx}\int_0^{3x} \cos(t)\, dt =$

a) $\sin(3x)$ b) $\cos(3x)$ c) $3\sin(3x)$

d) $3\cos(3x)$ e) none of these

27) $\int_0^1 \sqrt{x^2 - 2x + 1}\ dx$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) 4 e) none of these

28) The positive value of c, for x, that satisfies the conclusion of the Mean Value Theorem for Derivatives for $f(x) = 3x^2 - 5x + 1$ on the interval [2, 5] is

a) 1 b) $\frac{11}{6}$ c) $\frac{23}{6}$ d) $\frac{7}{2}$ e) none of these

29) The radius of a sphere is decreasing at the rate of 2 centimeters per second. At the instance when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere ?

a) -108π b) -72π c) -48π d) -24π e) none of these

30) The point of inflection on the graph of $y = x^3 - 15x^2 + 33x + 100$ has the coordinates

a) (9, 0) b) (5, – 48) c) (9, – 89) d) (5, 15)

e) none of these

31) The instantaneous rate of change of the function $f(t) = \frac{t^3+1}{4t+1}$ at $t = -1$ is

a) $\frac{12}{9}$ b) $\frac{4}{9}$ c) $-\frac{4}{9}$ d) $-\frac{12}{9}$ e) none of these

32) If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

a) no n b) n even, only c) n odd, only d) all n e) none of these

33) Let f be the function defined by $f(x) = \begin{cases} cx+d, & \text{for } x \le 2 \\ x^2 - cx, & \text{for } x > 2 \end{cases}$ where c and d are constants. If f is differentiable at $x = 2$, then the value of $c+d$ is

a) -4 b) -2 c) 0 d) 4 e) none of these

34) The point at which $f(x) = \frac{x^2+x-2}{x^2+7x+10}$ has a removable discontinuity is

a) $(-2, -1)$ b) $(-2, 1)$ c) $(1, 1)$ d) $(1, -1)$

e) none of these

35) The point on the curve $x^2 + 2y = 0$ that is closest to the point $\left(0, \frac{1}{2}\right)$ occurs when y is

a) $\frac{1}{2}$ b) 0 c) $-\frac{1}{2}$ d) -1 e) none of these

36) $\int_0^{\frac{\pi}{4}} \tan^2 x\,dx =$

a) $1 - \frac{\pi}{4}$ b) $\frac{\pi}{4} - 1$ c) $\frac{\pi-4}{4}$ d) $\frac{4+\pi}{4}$ e) none of these

37) If f be a differentiable function with $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. The value of $g'(3)$ is

a) $-\frac{1}{2}$ b) $-\frac{1}{8}$ c) $\frac{1}{6}$ d) $\frac{1}{3}$ e) none of these

38) The value of c that satisfies Rolle's Theorem for $f(x) = 2x^4 - 16x$ on the interval [0, 2] is

a) 2 b) $2^{-\frac{1}{3}}$ c) $(-2)^{\frac{1}{3}}$ d) $2^{\frac{1}{3}}$ e) none of these

39) The curve $y = \frac{3x}{x+7}$ has an asymptote at

a) $x = 7$ b) $y = 3$ c) $y = -7$ d) $x = 3$

e) none of these

40) The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height are both increasing at the constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and thr radius is 6 centimeters ?

a) 12π b) 24π c) 54π d) 72π e) none of these